

Anomalies on orbifolds with gauge symmetry breaking

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Abstract

We consider the breaking of 5D SUSY $G = SU(N + K)$ gauge symmetry into $H = SU(N) \times SU(K) \times U(1)$ on an orbifold $S^1/(Z_2 \times Z'_2)$. There appear two independent fixed points: one respects the full bulk gauge symmetry G while the other contains only the unbroken gauge symmetry H . In the model with one bulk $(N + K)$ -plet, giving a K -plet as the zero mode, we show that localized non-abelian gauge anomalies appear at the fixed points: the unbroken(H) gauge anomalies are equally distributed on both fixed points and the $H - (G/H) - (G/H)$ mixed gauge anomalies contribute only at the fixed point with G symmetry. We also find that in the case with a brane \bar{K} -plet added, the theory with the unbroken gauge group can be consistent up to the introduction of a bulk non-abelian Chern-Simons term. Finally, we comment on the gravitational anomalies and the Fayet-Iliopoulos terms in our model.

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1 Introduction

Recently the orbifold unification models in the existence of extra dimensions have drawn much attention due to their simplicity in performing the gauge symmetry breaking and the doublet-triplet splitting at the same time. The unwanted zero modes appearing in the unification models are projected out by boundary conditions in the extra dimension, i.e, they get masses of order of the compactification scale. For instance, the Minimal Supersymmetric Standard Model(MSSM) fields were obtained in the 5D SUSY $SU(5)$ model where the extra dimension is compactified on a simple orbifold $S^1/(Z_2 \times Z'_2)$ [1, 2]. The idea was also taken in the model with the 5D $SU(3)$ electroweak unification at a TeV[3]. In the orbifold with gauge symmetry breaking, in general, in addition to the fixed point where the bulk gauge symmetry is operative, there exists a fixed point where only the unbroken gauge group is respected[2]: for instance, $G_{SM} = SU(3) \times SU(2) \times U(1)$ in the case with the 5D SUSY $SU(5)$ model on $S^1/(Z_2 \times Z'_2)$. Therefore, we can put a multiplet(so called a brane field) at that fixed point allowed by the representation of the unbroken gauge group. It has been shown that introducing incomplete multiplets at the fixed point on GUT orbifolds is phenomenologically viable or necessary in some cases[2, 3]. However, when we introduce both bulk and brane fields on orbifolds, we have to be careful about the gauge anomalies localized on the boundaries[5, 6, 7, 4, 8].

In the 5D gauge theories with a $U(1)$ gauge factor on S^1/Z_2 , it has been shown that the abelian gauge anomalies coming from a bulk fermion are equally distributed only at the fixed points[9]. In this case, the localized anomalies can be interpreted as the sum of the anomalies from a localized zero mode and a 5D Chern-Simons term[9, 10]. Therefore, we naturally get the anomaly-free theory with both a brane fermion and a bulk fermion up to the addition of a 5D Chern-Simons term. At this point, we can raise the same question on orbifolds with gauge symmetry breaking[4]: whether two 4D fermions in the opposite representations can be *asymmetrically* embedded on the orbifold without anomaly problem.

In this paper, we do the explicit calculation of the gauge anomalies in the case with the gauge symmetry breaking on orbifolds. For our purpose of a general application, we consider the 5D SUSY $G = SU(N + K)$ gauge theory on $S^1/(Z_2 \times Z'_2)$, which is reduced to the 4D SUSY $H = SU(N) \times SU(K) \times U(1)$ gauge theory at the zero mode level by orbifold boundary conditions. In this case, there are two fixed points with different gauge groups: the full gauge symmetry G at $y = 0$ and the unbroken gauge symmetry H at $y = \pi R/2$. In the existence of a bulk hypermultiplet in the fundamental representation of the bulk gauge group, we only leave a K -plet among the GUT multiplet as the zero mode by the boundary conditions consistent with the gauge symmetry breaking. Then, we obtain the localized non-abelian anomalies from a bulk fermion by decomposing the 5D fermions and gauge fields in terms of the bulk eigenmodes and using the standard results of 4D anomalies. As a result, the H gauge anomalies are equally distributed at the fixed points while the $H - (G/H) - (G/H)$ mixed gauge anomalies are located only at $y = 0$.

In addition to the bulk field giving rise to a K -plet as the zero mode, we add a \bar{K} -plet at the fixed point $y = \pi R/2$. Then, the H gauge anomalies are localized with opposite signs at the fixed points while the other gauge anomalies remain nonzero at $y = 0$. Consequently, we show that all the remaining localized gauge anomalies are cancelled exactly by introducing a Chern-Simons 5-form with a jumping coefficient in the 5D action. The variation of this Chern-Simons term gives rise to the 4D gauge anomalies on the boundaries, which is exactly what is needed to cancel the gauge anomalies coming from the asymmetric assigning of two 4D fermions in the opposite representations under H .

Our paper is organized as follows. In the next section, we give an introduction to the gauge symmetry breaking on orbifolds by adopting a specific example, the 5D SUSY $SU(N + K)$ gauge theory on $S^1/(Z_2 \times Z'_2)$. Then, in the section 3, for this GUT orbifold, we derive the detail expression for the localized non-abelian anomalies coming from a bulk fermion in the fundamental representation of $SU(N + K)$. The section 4 is devoted to the localization problem of a bulk fermion and the cancellation of the localized gauge anomalies coming from a 4D anomaly-free combination of bulk and brane fermions. We also comment on the gravitational mixed anomalies and the Fayet-Iliopoulos terms in our model. Then, we conclude the paper in the last section.

2 Orbifold breaking of gauge symmetry

Let us consider the five-dimensional SUSY $G = SU(N + K)$ gauge theory compactified on an $S^1/(Z_2 \times Z'_2)$ orbifold. The fifth dimensional coordinate y is compactified to a circle $2\pi R \equiv 0$. Furthermore, the point $y = -a$ is identified to $y = a$ (Z_2 symmetry) and the point $y = (\pi R/2) + a$ is identified to $y = (\pi R/2) - a$ (Z'_2 symmetry). Then, the fundamental region of the extra dimension becomes the interval $[0, \frac{\pi R}{2}]$ between two fixed points $y = 0$ and $y = \frac{\pi R}{2}$.

For the two Z_2 symmetries, one can define their actions P and P' within the configuration space of any bulk field:

$$\phi(x, y) \rightarrow \phi(x, -y) = P\phi(x, y), \quad (1)$$

$$\phi(x, y') \rightarrow \phi(x, -y') = P'\phi(x, y'). \quad (2)$$

The (P, P') actions can involve all the symmetries of the bulk theory, for instance, the gauge symmetry and the R-symmetry in the supersymmetric case. In general, then, any bulk field ϕ can take one of four different Fourier expansions depending on their pair of two Z_2 parities, (i, j) as

$$\phi_{++} = \sum_{n=0}^{\infty} \sqrt{\frac{4}{2\delta_{n,0}\pi R}} \phi_{++}^{(2n)}(x^\mu) \cos \frac{2ny}{R} \quad (3)$$

$$\phi_{+-} = \sum_{n=0}^{\infty} \sqrt{\frac{4}{\pi R}} \phi_{+-}^{(2n+1)}(x^\mu) \cos \frac{(2n+1)y}{R} \quad (4)$$

$$\phi_{-+} = \sum_{n=0}^{\infty} \sqrt{\frac{4}{\pi R}} \phi_{-+}^{(2n+1)}(x^\mu) \sin \frac{(2n+1)y}{R} \quad (5)$$

$$\phi_{--} = \sum_{n=0}^{\infty} \sqrt{\frac{4}{\pi R}} i \phi_{--}^{(2n+2)}(x^\mu) \sin \frac{(2n+2)y}{R} \quad (6)$$

where x^μ is the 4D space-time coordinate.

The minimal supersymmetry in 5D corresponds to N=2 supersymmetry(or 8 supercharges) in the 4D N=1 language. Thus, a 5D chiral multiplet corresponds to an N=2 hypermultiplet consisting of two N=1 chiral multiplets with opposite charges. Two 4D Weyl spinors make

up one 5D spinor. On the other hand, a 5D vector multiplet corresponds to an N=2 vector multiplet composed of one N=1 vector multiplet ($V = (A_\mu, \lambda) \equiv V^q T^q$) and one N=1 chiral multiplet ($\Sigma = (\Phi + iA_5, \lambda') \equiv \Sigma^q T^q$). Upon compactification, we consider the case where one Z_2 breaks N=2 supersymmetry to N=1 while the other Z_2 breaks the bulk $G = SU(N + K)$ gauge group to its subgroup $H = SU(N) \times SU(K) \times U(1)$.

For instance, a bulk chiral multiplet $N + K$, which is composed of two chiral multiplets with opposite charges, $H(N + K) = (H_1, H_2)^T$ and $\tilde{H}(\overline{(N + K)}) = (\tilde{H}_1, \tilde{H}_2)^T$, transforms under Z_2 and Z'_2 identifications as

$$H(x, -y) = \eta P H(x, y), \quad \tilde{H}(x, -y) = -\eta P \tilde{H}(x, y) \quad (7)$$

$$H(x, -y') = \eta' P' H(x, y'), \quad \tilde{H}(x, -y') = -\eta' P' \tilde{H}(x, y') \quad (8)$$

where $y' \equiv y + \pi R/2$, and both η and η' can take +1 or -1. Then, choosing the parity matrices as

$$P = I_{N+K}, \quad P' = \text{diag}(-I_N, I_K), \quad (9)$$

where I_{N+K} is the $(N + K) \times (N + K)$ identity matrix and etc., and with $\eta = \eta' = 1$, the corresponding N=1 supermultiplets are split as follows

$$H_1^{(2n)} : \quad [(++); (1, K, \frac{1}{K})], \quad \text{mass} = 2n/R \quad (10)$$

$$H_2^{(2n+1)} : \quad [(+-); (N, 1, -\frac{1}{N})], \quad \text{mass} = (2n + 1)/R \quad (11)$$

$$\tilde{H}_1^{(2n+1)} : \quad [(-+); (\overline{N}, 1, \frac{1}{\overline{N}})], \quad \text{mass} = (2n + 1)/R \quad (12)$$

$$\tilde{H}_2^{(2n+2)} : \quad [(- -); (1, \overline{K}, -\frac{1}{\overline{K}})], \quad \text{mass} = (2n + 2)/R \quad (13)$$

where the brackets [] contain the quantum numbers of $Z_2 \times Z'_2 \times SU(N) \times SU(K) \times U(1)$. Consequently, upon compactification, there appears a zero mode only from the K -plet among the bulk field components while other fields get massive.

On the other hand, the bulk gauge multiplet is transformed under the two Z_2 transformations respectively as

$$V(x, -y) = P V(x, y) P^{-1}, \quad (14)$$

$$\Sigma(x, -y) = -P \Sigma(x, y) P^{-1}, \quad (15)$$

$$V(x, -y') = P' V(x, y') P'^{-1}, \quad (16)$$

$$\Sigma(x, -y') = -P' \Sigma(x, y') P'^{-1}. \quad (17)$$

Therefore, with the choice for the parity matrices in the fundamental representation as Eq. (9), the $G = SU(N + K)$ gauge symmetry is broken down to $H = SU(N) \times SU(K) \times U(1)$ because P' does not commute with all the gauge generators of $SU(N + K)$: $P' T^a P'^{-1} = T^a$ and $P' T^{\hat{a}} P'^{-1} = -T^{\hat{a}}$ where $q = (a, \hat{a})$ denote unbroken and broken generators, respectively. Actually, due to the orbifold boundary conditions for the gauge fields, we get the Z'_2 grading of $SU(N + K)$ as

$$[T^a, T^b] = i f^{abc} T^c, \quad [T^a, T^{\hat{b}}] = i f^{a\hat{b}c} T^c, \quad [T^{\hat{a}}, T^{\hat{b}}] = i f^{\hat{a}\hat{b}c} T^c \quad (18)$$

where f^{abc} and $f^{\hat{a}\hat{b}\hat{c}}$ is set to be zero for the Z'_2 invariance. As will be shown in the next section, the gauge anomalies in our orbifold model respect this group structure. It is interesting to see that this Z'_2 graded algebra also appears in the case with the spontaneous breaking of the $SU(N+K)$ global symmetry.

Consequently, upon compactification, the gauge multiplets of $SU(N+K)$ are

$$V^{a(n)} : [(++); (N^2 - 1, 1) + (1, K^2 - 1) + (1, 1)], \text{ mass} = 2n/R \quad (19)$$

$$V^{\hat{a}(2n+1)} : [(+-); (N, K) + (\overline{N}, \overline{K})], \text{ mass} = (2n+1)/R \quad (20)$$

$$\Sigma^{\hat{a}(2n+1)} : [(-+); (N, K) + (\overline{N}, \overline{K})], \text{ mass} = (2n+1)/R \quad (21)$$

$$\Sigma^{a(2n+2)} : [(- -); (N^2 - 1, 1) + (1, K^2 - 1) + (1, 1)], \text{ mass} = (2n+2)/R \quad (22)$$

where the brackets $[\]$ contain the quantum numbers of $Z_2 \times Z'_2 \times SU(N) \times SU(K)$. Therefore, the orbifolding retains only the $SU(N) \times SU(K) \times U(1)$ gauge multiplets as massless modes $V^{a(0)}$ while the KK massive modes for unbroken and broken gauge bosons are paired up separately. Here we make an interesting observation from our parity assignments that the $G = SU(N+K)$ gauge symmetry is fully conserved at $y = 0$ while only the unbroken gauge group $H = SU(N) \times SU(K) \times U(1)$ is operative at $y = \pi R/2$. Therefore, upon the orbifold compactification, it is possible to put some incomplete multiplets transforming only under the local gauge group at $y = \pi R/2$. Actually, since the parity conservation is assumed in the Lagrangian, each component of a gauge parameter $\omega = \omega^q T^q$ has the same Z_2 parities as those of the corresponding gauge field. Therefore, in the existence of the two Z_2 symmetries, the bulk gauge transformation does not respect the full $SU(N+K)$ gauge transformation but it is restricted as follows

$$\delta A_M^a = \partial_M \omega^a + i f^{abc} A_M^b \omega^c + i f^{\hat{a}\hat{b}\hat{c}} A_M^{\hat{b}} \omega^{\hat{c}}, \quad (23)$$

$$\delta A_M^{\hat{a}} = \partial_M \omega^{\hat{a}} + i f^{\hat{a}\hat{b}\hat{c}} A_M^{\hat{b}} \omega^{\hat{c}} + i f^{abc} A_M^b \omega^c \quad (24)$$

which are consistent with the Z'_2 graded algebra, eq. (18). Particularly, since $\omega^{\hat{a}}$ takes the same parities $(+, -)$ as $A_M^{\hat{a}}$, the gauge transformation at $y = \pi R/2$ becomes the one of the unbroken gauge group H from eq. (23).

3 Non-abelian anomalies on orbifolds with gauge symmetry breaking

A 5D fermion is not chiral in the 4D language. However, after orbifold compactification of the extra dimension, a chiral fermion can be obtained as the zero mode of a bulk non-chiral fermion. Then, the chiral fermion gives rise to the 4D gauge anomaly after integrating out the extra dimension. For the case with the 5D $U(1)$ gauge theory on S^1/Z_2 [9] or $S^1/(Z_2 \times Z'_2)$ [5], it was shown that the 4D gauge anomaly coming from a zero mode is equally distributed at the fixed points. In this section, we do the anomaly analysis in the case with the 5D $SU(N+K)$ gauge theory compactified on our gauge symmetry breaking orbifold, $S^1/(Z_2 \times Z'_2)$.

Let us consider a four-component bulk fermion in the fundamental representation of $SU(N+K)$. Then, the action is

$$S = \int d^4x \int_0^{2\pi R} dy \bar{\psi} (i \not{D} - \gamma_5 D_5 - m(y)) \psi \quad (25)$$

where $\not{D} = \gamma^\mu D_\mu$ and $D_M = \partial_M + i A_M$. Here $m(y)$ is a mass term for the bulk fermion and $A_M = A_M^q T^q$ is a classical non-abelian gauge field.

With the assignments of Z_2 and Z'_2 parities to a $(N+K)$ -plet hypermultiplet in the previous section, the fermion field transforms as

$$\psi(y) = \gamma_5 P \psi(-y), \quad \psi(y') = \gamma_5 P' \psi(-y') \quad (26)$$

where P and P' are given by Eq. (9), acting in the group space. Invariance of the action under two Z_2 's gives rise to the conditions for the mass function

$$m(y) = -m(-y), \quad m(y') = -m(-y'). \quad (27)$$

And the gauge fields also transform under Z_2 as

$$A_\mu(y) = P A_\mu(-y) P^{-1}, \quad A_5(y) = -P A_5(-y) P^{-1}, \quad (28)$$

and we replace $(y \rightarrow y', P \rightarrow P')$ for Z'_2 action.

Then, with $\psi = \psi^1 + \psi^2$, where 1 and 2 denotes K -plet and N -plet components respectively, the fermion field is decomposed into four independent chiral components

$$\psi^1 = \psi_L^1 + \psi_R^1, \quad \psi^2 = \psi_L^2 + \psi_R^2 \quad (29)$$

where

$$\gamma_5 \psi_{L(R)}^1 = \pm \psi_{L(R)}^1, \quad \gamma_5 \psi_{L(R)}^2 = \pm \psi_{L(R)}^2. \quad (30)$$

Due to the parity assignments, i.e., (\pm, \pm) for $\psi_{L(R)}^1$ and (\pm, \mp) for $\psi_{L(R)}^2$, we can expand each Weyl fermion in terms of KK modes

$$\psi_{L(R)}^1(x, y) = \sum_n \psi_{L(R)n}^1(x) \xi_n^{(\pm\pm)}(y), \quad (31)$$

$$\psi_{L(R)}^2(x, y) = \sum_n \psi_{L(R)n}^2(x) \xi_n^{(\pm\mp)}(y), \quad (32)$$

with

$$(-\partial_5 + m(y))(\partial_5 + m(y)) \xi_n^{(\pm+)}(y) = M_n^2 \xi_n^{(\pm+)}(y), \quad (33)$$

$$(\partial_5 + m(y))(-\partial_5 + m(y)) \xi_n^{(\mp-)}(y) = M_n^2 \xi_n^{(\mp-)}(y) \quad (34)$$

where M_n is the n th KK mass. Here we note that ξ 's make an orthonormal basis for the function on $[0, 2\pi R)$:

$$\int_0^{2\pi R} dy \xi_m^{(\pm\pm)}(y) \xi_n^{(\pm\pm)}(y) = \int_0^{2\pi R} dy \xi_m^{(\pm\mp)}(y) \xi_n^{(\pm\mp)}(y) = \delta_{mn}, \quad (35)$$

$$\int_0^{2\pi R} dy \xi_m^{(++)}(y) \xi_n^{(\pm\pm)}(y) = \int_0^{2\pi R} dy \xi_m^{(--)}(y) \xi_n^{(\pm\mp)}(y) = 0. \quad (36)$$

Under the gauge $A_5 = 0^2$, inserting the mode sum of the fermion into the 5D action, we obtain

$$\begin{aligned} S = & \int d^4x \left[\sum_n \bar{\psi}_n^1 (i\cancel{\partial} - M_{2n}) \psi_n^1 + \sum_n \bar{\psi}_n^2 (i\cancel{\partial} - M_{2n-1}) \psi_n^2 \right. \\ & \left. - \sum_{m,n} \left(V_{mn}(A^a) + V_{mn}(A^i) + V_{mn}(B) + V_{mn}(A^{\hat{a}}) \right) \right] \end{aligned} \quad (37)$$

²The result will be not changed in the case without a gauge condition[7]

where $\psi_n^1 = \psi_{Ln}^1 + \psi_{Rn}^1$ for $n > 0$ ($\psi_0^1 = \psi_{L0}^1$), $\psi_n^2 = \psi_{Ln}^2 + \psi_{Rn}^2$, and V_{mn} 's denote gauge vertex couplings. The $G = SU(N + K)$ gauge fields ($A = A^q T^q$) can be decomposed into

$$(N + K)^2 - 1 \rightarrow (N^2 - 1, 1) + (1, K^2 - 1) + (1, 1) + (N, K) + (\bar{N}, \bar{K}), \quad (38)$$

that is, $A^a T^a (a = 1, \dots, N^2 - 1)$, $A^i T^i (i = 1, \dots, K^2 - 1)$, $A^{(N+K)^2-1} T^{(N+K)^2-1} \equiv BT^B$ gauge fields for the $H = SU(N) \times SU(K) \times U(1)$ group, and $A^{\hat{a}}(t^{\hat{a}})_{\alpha r} \equiv X^{\alpha r} (\alpha = 1, \dots, N; r = 1, \dots, K)$ gauge fields for the G/H group, respectively. Here, broken group generators are related to $t^{\hat{a}}$ as

$$T^{\hat{a}} \equiv \begin{pmatrix} 0 & t^{\hat{a}} \\ (t^{\hat{a}})^\dagger & 0 \end{pmatrix}. \quad (39)$$

Then, V_{mn} 's are given by the following:

$$\begin{aligned} V_{mn}(A^a) &= J_{mn(+)}^{\mu a} \mathcal{A}_{mn\mu}^{a(+)} + J_{mn(-)}^{\mu a} \mathcal{A}_{mn\mu}^{a(-)} \\ V_{mn}(A^i) &= J_{mn(+)}^{\mu i} \mathcal{A}_{mn\mu}^{i(+)} + J_{mn(-)}^{\mu i} \mathcal{A}_{mn\mu}^{i(-)} \\ V_{mn}(B) &= J_{mn(+)}^{\mu B} \mathcal{B}_{mn\mu}^{(+)} + J_{mn(-)}^{\mu B} \mathcal{B}_{mn\mu}^{(-)} + J_{mn(+)}^{\mu B} \mathcal{B}_{mn\mu}^{(+)} + J_{mn(-)}^{\mu B} \mathcal{B}_{mn\mu}^{(-)} \\ V_{mn}(A^{\hat{a}}) &= J_{mn(+)}^{\mu \hat{a}} \mathcal{A}_{mn\mu}^{\hat{a}(+)} + J_{mn(-)}^{\mu \hat{a}} \mathcal{A}_{mn\mu}^{\hat{a}(-)} \end{aligned} \quad (40)$$

where

$$\mathcal{A}_{mn\mu}^{a(\pm)} = \int_0^{2\pi R} dy \xi_m^{(\pm)}(y) \xi_n^{(\pm)}(y) A_\mu^a(x, y), \quad (41)$$

$$\mathcal{A}_{mn\mu}^{i(\pm)} = \int_0^{2\pi R} dy \xi_m^{(\pm)}(y) \xi_n^{(\pm)}(y) A_\mu^i(x, y), \quad (42)$$

$$\mathcal{B}_{mn\mu}^{(\pm)} = \int_0^{2\pi R} dy \xi_m^{(\pm)}(y) \xi_n^{(\pm)}(y) B_\mu(x, y), \quad (43)$$

$$\mathcal{B}_{mn\mu}^{(\pm)} = \int_0^{2\pi R} dy \xi_m^{(\pm)}(y) \xi_n^{(\pm)}(y) B_\mu(x, y), \quad (44)$$

$$\mathcal{A}_{mn\mu}^{\hat{a}(\pm)} = \int_0^{2\pi R} dy \xi_m^{(\pm)}(y) \xi_n^{(\pm)}(y) A_\mu^{\hat{a}}(x, y) \quad (45)$$

and

$$J_{mn(\pm)}^{\mu a} = \overline{\psi}_m^2 \gamma^\mu P_\pm T^a \psi_n^2, \quad J_{mn(\pm)}^{\mu i} = \overline{\psi}_m^1 \gamma^\mu P_\pm T^i \psi_n^1, \quad (46)$$

$$J_{mn(\pm)}^{\mu B} = \overline{\psi}_m^1 \gamma^\mu P_\pm T_{K \times K}^B \psi_n^1, \quad J_{mn(\pm)}^{\mu B} = \overline{\psi}_m^2 \gamma^\mu P_\pm T_{N \times N}^B \psi_n^2, \quad (47)$$

$$J_{mn(\pm)}^{\mu \hat{a}} = \overline{\psi}_m^2 \gamma^\mu P_\pm t^{\hat{a}} \psi_n^1 + \overline{\psi}_m^1 \gamma^\mu P_\pm (t^{\hat{a}})^\dagger \psi_n^2 \quad (48)$$

with $P_\pm = (1 \pm \gamma_5)/2$. Here a decomposition of T^B is understood such as $T^B = \text{diag.}(T_{N \times N}^B, T_{K \times K}^B)$. We note that the chiral current for the $SU(N + M)$ gauge symmetry is split into chiral currents coupled to the unbroken and broken gauge fields.

Before going into the detail calculation of the gauge anomalies, we find that there is the selection rule for the possible gauge anomalies due to the parity conservation: only the gauge anomalies of the type AAA and AXX are present, but neither AAX or XXX . (Here A denote

the remaining gauge fields with $(+, +)$ while X denotes the broken gauge fields with $(+, -)$, respectively.) Therefore, gauge anomalies coming from a bulk fermion correspond to some part of the full $SU(N + K)$ gauge anomalies, which reflects the Z'_2 grading of the bulk gauge group (18).

Applying the classical equations of motion and the standard results for the 4D chiral anomalies[9, 6, 7], we can derive the anomalies for the chiral currents classified above. By making an inverse Fourier-transformation by the convolution of the bulk eigenmodes, the 5D gauge vector current $J^{Mq} = \bar{\psi}\gamma^M T^q \psi$ is given by

$$J^{\mu a}(x, y) = \sum_{m,n} (\xi_m^{(+ -)} \xi_n^{(+ -)} J_{mn(+ -)}^{\mu a} + \xi_m^{(- +)} \xi_n^{(- +)} J_{mn(- +)}^{\mu a}), \quad (49)$$

$$J^{\mu i}(x, y) = \sum_{m,n} (\xi_m^{(+ +)} \xi_n^{(+ +)} J_{mn(+ +)}^{\mu i} + \xi_m^{(- -)} \xi_n^{(- -)} J_{mn(- -)}^{\mu i}), \quad (50)$$

$$J^{\mu B}(x, y) = \sum_{m,n} (\xi_m^{(+ +)} \xi_n^{(+ +)} J_{mn(+ +)}^{\mu B} + \xi_m^{(- -)} \xi_n^{(- -)} J_{mn(- -)}^{\mu B} + \xi_m^{(+ -)} \xi_n^{(+ -)} J_{mn(+ -)}^{\mu B} + \xi_m^{(- +)} \xi_n^{(- +)} J_{mn(- +)}^{\mu B}), \quad (51)$$

$$J^{\mu \hat{a}}(x, y) = \sum_{m,n} (\xi_m^{(+ -)} \xi_n^{(+ +)} J_{mn(+)}^{\mu \hat{a}} + \xi_m^{(- +)} \xi_n^{(- -)} J_{mn(-)}^{\mu \hat{a}}) \quad (52)$$

and we replace $(\mu \rightarrow 5)$ for J^{5q} . Consequently, it turns out that the divergence of the 5D gauge vector current is given in terms of the 4D gauge anomalies as

$$(D_M J^M)^a(x, y) = f_2(y)(\mathcal{Q}^a(A) + \mathcal{Q}^a(X)), \quad (53)$$

$$(D_M J^M)^i(x, y) = f_1(y)(\mathcal{Q}^i(A) + \mathcal{Q}^i(X)), \quad (54)$$

$$(D_M J^M)^B(x, y) = f_1(y)(\mathcal{Q}_+^B(A) + \mathcal{Q}_+^B(X)) + f_2(y)(\mathcal{Q}_-^B(A) + \mathcal{Q}_-^B(X)), \quad (55)$$

$$(D_M J^M)^{\hat{a}}(x, y) = f_1(y)(\mathcal{Q}_1^{\hat{a}}(X) + \mathcal{Q}_+^{\hat{a}}(X)) + f_2(y)(\mathcal{Q}_2^{\hat{a}}(X) + \mathcal{Q}_-^{\hat{a}}(X)) \quad (56)$$

where

$$f_1(y) = \sum_n \left[(\xi_n^{(+ +)}(y))^2 - (\xi_n^{(- -)}(y))^2 \right] = \frac{1}{4} \sum_n \delta(y - \frac{n\pi R}{2}), \quad (57)$$

$$f_2(y) = \sum_n \left[(\xi_n^{(+ -)}(y))^2 - (\xi_n^{(- +)}(y))^2 \right] = \frac{1}{4} \sum_n (-1)^n \delta(y - \frac{n\pi R}{2}). \quad (58)$$

The localized gauge anomalies \mathcal{Q} 's are composed of two large parts: anomalies for unbroken group components and broken group components of the 5D vector current. The anomalies for

unbroken group components involve not only unbroken gauge fields

$$\mathcal{Q}^a(A) = \frac{1}{32\pi^2}(D^{abc}F_{\mu\nu}^b\tilde{F}^{c\mu\nu}(x,y) + D^{abB}F_{\mu\nu}^b\tilde{F}^{B\mu\nu}(x,y)), \quad (59)$$

$$\mathcal{Q}^i(A) = \frac{1}{32\pi^2}D^{ijB}F_{\mu\nu}^j\tilde{F}^{B\mu\nu}(x,y), \quad (60)$$

$$\begin{aligned} \mathcal{Q}_+^B(A) &= \frac{1}{32\pi^2}\text{Tr}(T_{K\times K}^B)^3F_{\mu\nu}^B\tilde{F}^{B\mu\nu}(x,y) \\ &+ \frac{1}{64\pi^2}\text{Tr}(\{T_{K\times K}^B, T^i\}T^j)F_{\mu\nu}^i\tilde{F}^{j\mu\nu}(x,y), \end{aligned} \quad (61)$$

$$\begin{aligned} \mathcal{Q}_-^B(A) &= \frac{1}{32\pi^2}\text{Tr}(T_{N\times N}^B)^3F_{\mu\nu}^B\tilde{F}^{B\mu\nu}(x,y) \\ &+ \frac{1}{64\pi^2}\text{Tr}(\{T_{N\times N}^B, T^a\}T^b)F_{\mu\nu}^a\tilde{F}^{b\mu\nu}(x,y), \end{aligned} \quad (62)$$

$$\begin{aligned} \mathcal{Q}_+^B(A) + \mathcal{Q}_-^B(A) &= \frac{1}{32\pi^2}(D^{BBB}F_{\mu\nu}^B\tilde{F}^{B\mu\nu}(x,y) + D^{Bij}F_{\mu\nu}^i\tilde{F}^{j\mu\nu}(x,y) \\ &+ D^{Bab}F_{\mu\nu}^a\tilde{F}^{b\mu\nu}(x,y)) \equiv \mathcal{Q}^B(A), \end{aligned} \quad (63)$$

but also broken gauge fields

$$\mathcal{Q}^a(X) = \frac{1}{64\pi^2}\text{Tr}(T^a t^{\hat{b}}(t^{\hat{c}})^\dagger)F_{\mu\nu}^{\hat{b}}\tilde{F}^{\hat{c}\mu\nu} = \frac{1}{32\pi^2}D^{\hat{a}\hat{b}\hat{c}}F_{\mu\nu}^{\hat{b}}\tilde{F}^{\hat{c}\mu\nu}, \quad (64)$$

$$\mathcal{Q}^i(X) = \frac{1}{64\pi^2}\text{Tr}(T^i(t^{\hat{b}})^\dagger t^{\hat{c}})F_{\mu\nu}^{\hat{b}}\tilde{F}^{\hat{c}\mu\nu} = \frac{1}{32\pi^2}D^{i\hat{b}\hat{c}}F_{\mu\nu}^{\hat{b}}\tilde{F}^{\hat{c}\mu\nu}, \quad (65)$$

$$\mathcal{Q}_+^B(X) = \frac{1}{64\pi^2}\text{Tr}(\{T^B, (t^{\hat{b}})^\dagger\}t^{\hat{c}})F_{\mu\nu}^{\hat{b}}\tilde{F}^{\hat{c}\mu\nu},$$

$$\mathcal{Q}_-^B(X) = \frac{1}{64\pi^2}\text{Tr}(\{T^B, t^{\hat{b}}\}(t^{\hat{c}})^\dagger)F_{\mu\nu}^{\hat{b}}\tilde{F}^{\hat{c}\mu\nu},$$

$$\mathcal{Q}_+^B(X) + \mathcal{Q}_-^B(X) = \frac{1}{32\pi^2}D^{B\hat{b}\hat{c}}F_{\mu\nu}^{\hat{b}}\tilde{F}^{\hat{c}\mu\nu} \equiv \mathcal{Q}^B(X). \quad (66)$$

On the other hand, the anomalies for broken group components of the 5D vector current become

$$\begin{aligned} \mathcal{Q}_1^{\hat{a}}(X) &= \frac{1}{64\pi^2}\text{Tr}(\{t^{\hat{a}}, (t^{\hat{a}})^\dagger\} + \{(t^{\hat{a}})^\dagger, t^{\hat{b}}\})T^a F_{\mu\nu}^{\hat{a}}\tilde{F}^{a\mu\nu} \\ &= \frac{1}{32\pi^2}D^{\hat{a}\hat{b}a}F_{\mu\nu}^{\hat{a}}\tilde{F}^{a\mu\nu}, \end{aligned} \quad (67)$$

$$\begin{aligned} \mathcal{Q}_2^{\hat{a}}(X) &= \frac{1}{64\pi^2}\text{Tr}(\{t^{\hat{a}}, (t^{\hat{a}})^\dagger\} + \{(t^{\hat{a}})^\dagger, t^{\hat{b}}\})T^i F_{\mu\nu}^{\hat{a}}\tilde{F}^{i\mu\nu} \\ &= \frac{1}{32\pi^2}D^{\hat{a}\hat{b}i}F_{\mu\nu}^{\hat{a}}\tilde{F}^{i\mu\nu}, \end{aligned} \quad (68)$$

$$\begin{aligned} \mathcal{Q}_+^{\hat{a}}(X) &= \frac{1}{64\pi^2}\text{Tr}((t^{\hat{a}})^\dagger t^{\hat{b}} + (t^{\hat{b}})^\dagger t^{\hat{a}})T_{K\times K}^B F_{\mu\nu}^{\hat{a}}\tilde{F}^{B\mu\nu} \\ &= \frac{1}{64\pi^2}\text{Tr}(\{T^{\hat{a}}, T^{\hat{b}}\}T_{K\times K}^B)F_{\mu\nu}^{\hat{a}}\tilde{F}^{B\mu\nu}, \end{aligned} \quad (69)$$

$$\begin{aligned} \mathcal{Q}_-^{\hat{a}}(X) &= \frac{1}{64\pi^2}\text{Tr}((t^{\hat{a}}(t^{\hat{b}})^\dagger + t^{\hat{b}}(t^{\hat{a}})^\dagger)T_{N\times N}^B)F_{\mu\nu}^{\hat{a}}\tilde{F}^{B\mu\nu} \\ &= \frac{1}{64\pi^2}\text{Tr}(\{T^{\hat{a}}, T^{\hat{b}}\}T_{N\times N}^B)F_{\mu\nu}^{\hat{a}}\tilde{F}^{B\mu\nu}, \end{aligned} \quad (70)$$

$$\mathcal{Q}_+^{\hat{a}}(X) + \mathcal{Q}_-^{\hat{a}}(X) = \frac{1}{32\pi^2}D^{\hat{a}\hat{b}B}F_{\mu\nu}^{\hat{a}}\tilde{F}^{B\mu\nu} \equiv \mathcal{Q}_3^{\hat{a}}(X) \quad (71)$$

In all the expressions for the anomalies above, we note that D^{abc} denotes the symmetrized trace of group generators

$$D^{abc} = \frac{1}{2} \text{Tr}(\{T^a, T^b\}T^c) \quad (72)$$

and other D symbols with different group indices are similarly understood.

As a result, we find that a bulk fermion gives rise to the localized gauge anomalies for all gauge components of the 5D vector current. Moreover, since the broken gauge fields vanish at $y = \pi R/2$ due to their boundary conditions, the localized gauge anomalies at $y = \pi R/2$ are only $\mathcal{Q}(A)$'s, i.e., the H gauge anomalies. However, at the other fixed point $y = 0$, in addition to $\mathcal{Q}(A)$'s, there also appear the localized gauge anomalies $\mathcal{Q}(X)$'s, i.e., the $H - (G/H) - (G/H)$ gauge anomalies. With this in mind and restricting to the region $[0, 2\pi R]$, we can rewrite the divergence of the 5D vector current as

$$(D_M J^M)^a(x, y) = \frac{1}{2} \left(\delta(y) - \delta(y - \frac{\pi R}{2}) \right) \mathcal{Q}^a(A) + \frac{1}{2} \delta(y) \mathcal{Q}^a(X), \quad (73)$$

$$(D_M J^M)^i(x, y) = \frac{1}{2} \left(\delta(y) + \delta(y - \frac{\pi R}{2}) \right) \mathcal{Q}^i(A) + \frac{1}{2} \delta(y) \mathcal{Q}^i(X), \quad (74)$$

$$\begin{aligned} (D_M J^M)^B(x, y) &= \frac{1}{2} \left(\delta(y) + \delta(y - \frac{\pi R}{2}) \right) \mathcal{Q}_+^B(A) + \frac{1}{2} \left(\delta(y) - \delta(y - \frac{\pi R}{2}) \right) \mathcal{Q}_-^B(A) \\ &\quad + \frac{1}{2} \delta(y) \mathcal{Q}^B(X), \end{aligned} \quad (75)$$

$$(D_M J^M)^{\hat{a}}(x, y) = \frac{1}{2} \delta(y) \mathcal{Q}^{\hat{a}}(X) \quad (76)$$

where $\mathcal{Q}^{\hat{a}}(X) \equiv \mathcal{Q}_1^{\hat{a}}(X) + \mathcal{Q}_2^{\hat{a}}(X) + \mathcal{Q}_3^{\hat{a}}(X)$.

4 Localization of a bulk field and anomaly problem

As shown in the section 2, we can freely put some brane fields consistently with the local gauge symmetries at the fixed points: a brane field at $y = 0$ should be a representation of $SU(N + K)$ while a brane field at $y = \pi R/2$ should be a representation of $SU(N) \times SU(K) \times U(1)$. Since we assume that a bulk fermion gives rise to a K -plet as the zero mode and we want to have the anomaly-free theory at least at the zero mode level, we can only put a brane field of \bar{K} -plet at $y = \pi R/2$. This introduction of an incomplete brane multiplet is sufficient for the 4D anomaly-free theory at low energies but it could be inconsistent due to the existence of the localized gauge anomalies on the boundaries of the extra dimension. In this section, we consider the localization of a bulk fermion with a kink mass and subsequently deal with the appearing anomaly problem by using the results in the previous section.

It was shown in the literature that the localization of a bulk fermion can be realized by introducing a kink mass in the Lagrangian and even a brane fermion is possible in the limit of a kink mass being infinite[9]. In the 5D $U(1)$ gauge theory on S^1/Z_2 with a single bulk fermion, as a result of introducing an infinite kink mass, the anomaly contribution from a bulk fermion on the boundaries of the extra dimension was interpreted as the sum of contributions from a brane fermion and a parity-violating Chern-Simon term in 5D[9]. In other words, as a kink mass becomes infinite, heavy KK modes are decoupled but their effects remain as a local counterterm such as the 5D Chern-Simon term. The similar observation has been made for the non-abelian anomalies on orbifolds[4].

In our case with gauge symmetry breaking on orbifolds, however, we should be careful about the sign of a kink mass because an infinite kink mass could give rise to the localization of the unwanted bulk modes as a massless mode[7, 11, 12]. For instance, both $(+, +)$ and $(+, -)((-, +))$ modes with positively (negatively) infinite kink masses could be massless and localized at $y = 0(y = \pi R/2)$. Suppose that there are the universal(preserving the bulk gauge symmetry) kink masses for even and odd modes, i.e., $m(y) = M\epsilon(y)I_{(N+K)\times(N+K)}$ in eq. (25) where $\epsilon(y)$ is the sign function with periodicity πR . Then, for $M \rightarrow +\infty$, there appears a massless (N, K) multiplet from the bulk field, which is localized at $y = 0$. On the other hand, for $M \rightarrow -\infty$, a massless (\bar{N}, K) multiplet is localized at $y = \pi R/2$. Therefore, it seems difficult to realize an incomplete brane field from the 5D field theory itself without explicit breaking the bulk gauge symmetry of the Lagrangian. Nonetheless, there is a realistic example where it is indispensable to introduce the quark sector at the fixed point as an incomplete multiplet in the 5D $SU(3)$ electroweak unification model on $S^1/(Z_2 \times Z'_2)$ [3]. Thus, provided that the incomplete brane field is required for the consistent low energy particle physics, we find that introducing a 5D non-abelian Chern-Simons term makes the theory with the incomplete brane field anomaly-free[4].

When we introduce a brane \bar{K} -plet at $y = \pi R/2$, it gives rise to 4D gauge anomalies such as $-\mathcal{Q}^i(A)$ and $-\mathcal{Q}_+^B(A)$ at that fixed point. Therefore, with the addition of the brane \bar{K} -plet to a bulk $(N + K)$ -plet, the divergence of the 5D vector current is changed to

$$(D_M J^M)^a(x, y) = \frac{1}{2} \left(\delta(y) - \delta(y - \frac{\pi R}{2}) \right) \mathcal{Q}^a(A) + \frac{1}{2} \delta(y) \mathcal{Q}^a(X), \quad (77)$$

$$(D_M J^M)^i(x, y) = \frac{1}{2} \left(\delta(y) - \delta(y - \frac{\pi R}{2}) \right) \mathcal{Q}^i(A) + \frac{1}{2} \delta(y) \mathcal{Q}^i(X), \quad (78)$$

$$(D_M J^M)^B(x, y) = \frac{1}{2} \left(\delta(y) - \delta(y - \frac{\pi R}{2}) \right) \mathcal{Q}^B(A) + \frac{1}{2} \delta(y) \mathcal{Q}^B(X), \quad (79)$$

$$(D_M J^M)^{\hat{a}}(x, y) = \frac{1}{2} \delta(y) \mathcal{Q}^{\hat{a}}(X). \quad (80)$$

Here we observe that the total localized gauge anomalies only involving the unbroken gauge group($\mathcal{Q}(A)$'s) appear in the combination of $(\delta(y) - \delta(y - \pi R/2))$, so their integrated gauge anomalies vanish. On the other hand, the anomalies involving broken gauge fields($\mathcal{Q}(X)$'s) remain nonzero even after integration because $\mathcal{Q}(X)$'s vanish only at $y = \pi R/2$. This asymmetric localization of $\mathcal{Q}(X)$'s reflects the difference between two fixed point groups. The existence of the localized gauge anomalies could make the theory with the unbroken gauge group anomalous. However, these localized gauge anomalies can be exactly cancelled with the introduction of a Chern-Simons(CS) 5-form $Q_5[A = A^a T^a]$ with a jumping coefficient in the action[4]

$$\mathcal{L}_{CS} = -\frac{1}{96\pi^2} \epsilon(y) Q_5[A] \quad (81)$$

where $\epsilon(y)$ is the sign function with periodicity πR and

$$Q_5[A] = \text{Tr} \left(AdAdA + \frac{3}{2} A^3 dA + \frac{3}{5} A^5 \right). \quad (82)$$

The parity-odd function $\epsilon(y)$ in front of Q_5 is necessary for the parity invariance because Q_5 is a parity-odd quantity according to our parity assignments for bulk gauge fields, eqs. (19)-(22). Under the gauge transformation $\delta A = d\omega + [A, \omega] \equiv D\omega$,

$$\delta Q_5 = Q_4^1[\delta A, A] = \text{str} \left(D\omega d(AdA + \frac{1}{2} A^3) \right) \quad (83)$$

where str means the symmetrized trace and the restricted gauge transformation in eqs. (23) and (24) is understood. Then, due to the sign function in front of Q_5 , the variation of the Chern-Simons action gives rise to the 4D consistent anomalies on the boundaries

$$\begin{aligned}\delta\mathcal{L}_{CS} &= \frac{1}{48\pi^2}(\delta(y) - \delta(y - \frac{\pi R}{2}))\epsilon^{\mu\nu\rho\sigma} \sum_{q=a,i,B} \omega^q \text{str}(T^q \partial_\mu (A_\nu \partial_\rho A_\sigma + \frac{1}{2} A_\nu A_\rho A_\sigma)) \\ &+ \frac{1}{48\pi^2} \delta(y) \epsilon^{\mu\nu\rho\sigma} \omega^{\hat{a}} \text{str}(T^{\hat{a}} \partial_\mu (A_\nu \partial_\rho A_\sigma + \frac{1}{2} A_\nu A_\rho A_\sigma)).\end{aligned}\quad (84)$$

The consistent anomalies we obtained here can be changed to the covariant anomalies[13] by regarding the covariant non-abelian gauge current J_μ^q as being redefined from a non-covariant gauge current \tilde{J}_μ^q as

$$J_\mu^q(x, y) = \tilde{J}_\mu^q(x, y) + U_\mu^q(x, y) \quad (85)$$

where

$$\begin{aligned}U_\mu^{q=(a,i,B)} &= -\frac{1}{96\pi^2}(\delta(y) - \delta(y - \frac{\pi R}{2}))\epsilon^{\mu\nu\rho\sigma} \text{str}(T^q (A_\nu F_{\rho\sigma} + F_{\rho\sigma} A_\nu - A_\nu A_\rho A_\sigma)) \\ U_\mu^{q=\hat{a}} &= -\frac{1}{96\pi^2} \delta(y) \epsilon^{\mu\nu\rho\sigma} \text{str}(T^q (A_\nu F_{\rho\sigma} + F_{\rho\sigma} A_\nu - A_\nu A_\rho A_\sigma)).\end{aligned}\quad (86)$$

Consequently, when we take into account the fact that the broken gauge fields are vanishing at $y = \pi R/2$, the CS term contributes to the anomaly for the 5D covariant gauge current as

$$\begin{aligned}(D_M J^M)^{q_1=(a,i,B)} &= -\frac{1}{64\pi^2}(\delta(y) - \delta(y - \frac{\pi R}{2})) \sum_{q_2, q_3=(b,j,B)} \text{str}(T^{q_1} T^{q_2} T^{q_3}) F_{\mu\nu}^{q_2} F^{q_3\mu\nu} \\ &- \frac{1}{64\pi^2} \delta(y) \sum_{q_2, q_3=\hat{a}} \text{str}(T^{q_1} T^{q_2} T^{q_3}) F_{\mu\nu}^{q_2} F^{q_3\mu\nu},\end{aligned}\quad (87)$$

$$(D_M J^M)^{q_1=\hat{a}} = -\frac{1}{64\pi^2} \delta(y) \sum_{q_2 q_3=\hat{b}(a,i,B)} \text{str}(T^{q_1} T^{q_2} T^{q_3}) F_{\mu\nu}^{q_2} F^{q_3\mu\nu} \quad (88)$$

where $q_{1,2,3}$ run the bulk group indices. It turns out that the CS contributions to the anomalies exactly cancel the remaining localized covariant gauge anomalies on the boundaries, eq. (77)-(80).

Before closing this section, let us comment on the gravitational mixed anomalies and the Fayet-Iliopoulos(FI) terms[14, 8] for our set of bulk and brane fields. The only place where the $U(1)$ -graviton-graviton anomalies could appear is the fixed point $y = \pi R/2$ with the local gauge group including a $U(1)$ gauge factor. As argued in the literature[4], there is no gravitational counterpart $A \wedge R \wedge R$ of the 5D Chern-Simons term since the non-abelian gauge fields propagate in the bulk. It has been shown that the gravitational anomalies at $y = \pi R/2$ indeed cancel between the bulk and brane contributions without the need of a bulk Chern-Simons term[4]. Then, since both gravitational anomalies and FI terms are proportional to the common factor $\text{Tr}(q)$, where q is the $U(1)$ charge operator, it seems that the absence of the gravitational anomalies should guarantee the absence of the FI terms which could also exist at $y = \pi R/2$. This is the requirement for the stability of the 4D supersymmetric theory. The FI terms induced from a bulk field are composed of both the quadratically divergent part and the logarithmically divergent part[5, 7, 8, 11]. However, there is no contribution from a brane

field to the logarithmically divergent FI term. In this case, we observe that the bulk modes contribute to a nonzero log FI term localized at $y = \pi R/2$ but they do not affect either the 4D supersymmetry or the KK spectrum of the bulk field[8]. We hope to deal with the consistency with the log FI term in our future publication.

5 Conclusion

We considered the breaking of the 5D non-abelian gauge symmetry on $S^1/(Z_2 \times Z_2)$ orbifold. Then, we presented the localized gauge anomalies coming from a bulk fundamental field through the explicit KK mode decomposition of the 5D fields. In the orbifold with gauge symmetry breaking, there are fixed points with their own local gauge symmetries. Thus, there is the possibility of embedding some incomplete multiplets at the fixed point with unbroken gauge group, which can be sometimes phenomenologically preferred. Therefore, we considered the 4D anomaly combination of a brane field and a bulk zero mode, which even does not have localized gauge anomalies up to the addition of a Chern-Simon 5-form with some jumping coefficient. Finally, we made some comment on the $U(1)$ -gravitational mixed anomalies and the FI terms in our orbifold model.

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